**Model Test Paper**

# GLA UNIVERSITY, MATHURA

## (M.Sc. Mathematics)

Q.No.1.
If the rank of a (5 x 6) matrix $\emptyset$ is 4, then which one of the following statements is correct?
(A) Matrix $\emptyset$ will have four linearly independent rows and four linearly independent columns.
(B) Matrix $\emptyset$ will have four linearly indepent rows and five linearly independent columns.
(C) $\emptyset\,\emptyset^{T}$ will be invertible.
(D) $\emptyset^{T}\emptyset$ will be invertible.

Q. No. 2.
If one of the eigenvalues of $[A]_{n\times n}$ is zero, it implies that
(A) The solution to$[A][X] = [C]$ system of equations is unique.
(B) The determinant of $[A]$ is zero.
(C) The solution to $[A][X] = [0]$system of equations is trivial.
(D) The determinant of $[A]$ is nonzero.

Q. No. 3.
The necessary condition for maxima or minima of a function $f(x, y, z)$ is
(A) $\frac{\partial f}{\partial x} = 0;\ \frac{\partial f}{\partial y} < 0, \frac{\partial f}{\partial z} > 0.$
(B) $\frac{\partial f}{\partial x} < 0;\ \frac{\partial f}{\partial y} = \frac{\partial f}{\partial z} = 0.$
(C) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} > 0,\ \frac{\partial f}{\partial z} = 0.$
(D) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = \frac{\partial f}{\partial z} = 0.$

Q. No. 4.
For the differential equation $xdy - ydx = 0$, which of the following is an integrating factor?
(A) $\frac{1}{y^2} f\left(\frac{y}{x}\right)$
(B) $\frac{1}{x^2} f(xy)$
(C) $\frac{1}{x^2} f\left(\frac{y}{x}\right)$
(D) $f\left(\frac{y}{x}\right)$

Q. No. 5.
The asymptotes of the curve $\frac{a^3}{x^3} - \frac{b^3}{y^3} = 1$ are
(A) $x - a = 0, y + b = 0$
(B) $y = x + 1, y = -2x + 2$
(C) $x + a = 0, y - b = 0$
(D) None of these

Q.No.6.
For the differential equation $y\,dx - x\,dy = 0$, which one is integrating factor?
(A) $\frac{1}{x^2-y^2}$
(B) $\frac{1}{x^2+y^2}$
(C) $\frac{y}{x}$
(D) $\frac{x}{y}$

Q.No.7.
The coefficient of $x^3$ in the Taylor's expansion of the function $\cos x \cos y$ about (0, 0) is
(A) $1/3$
(B) 0
(C) $1/6$
(D) $1/2$

Q.No.8.
The derivative $f'(x)$ of $f(x)$ is negative or zero in the interval $(a, b)$ but not uniformly zero. Then
(A) $f(a) > f(b)$
(B) $f(b) > f(a)$
(C) $f(a) = f(b)$
(D) None of these

Q.No.9.
For the set X = {a, b}, which of the following is a topology?
(A) $\tau = [\emptyset,\ \{a\}, \{b\}]$
(B) $\tau = [\emptyset,\ \{a\}, \{b\}, \{a, b\}]$
(C) $\tau = [\{a\}, \{b\}, \{a, b\}]$
(D) None of these

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Q.No.10.
If two distinct points in a topological space can be separated by two disjoint open sets, then it is called

(A) Regular space
(B) Normal space
(C) Hausdorff space
(D) Completely regular space

Q.No.11.
The closure of the set X = (0, 1) ∪ (1, 2) is
(A) [0, 2] – {1}
(B) [0, 2]
(C) [0, 2)
(D) (0, 2]

Q.No.12.
If second and fourth moments about mean of a distribution are 3 and 26 respectively. Then the distribution is
(A) Leptokurtic
(B) Mesokurtic
(C) Platykurtic
(D) None of these

Q.No.13.
$E(X^2) - [E(X)]^2$ is
(A) $E(X)$
(B) $E(X^2)$
(C) $Var(X)$
(D) $S.D.(X)$

Q.No.14.
Which of the following equation is elliptic?
(A) Laplace equation
(B) Wave equation
(C) Heat Equation
(D) None of these

Q.No.15.
If A is the matrix of order m x n, then

(A) Rank (A) = max (m, n)
(B) $Rank\ (A) \leq min\ (m, n)$
(C) $Rank\ (A) = m$
(D) $Rank\ (A) = n$

Q.No.16.
A force field $\vec{F}$ is said to be conservative if

(A) curl $\vec{F} = \vec{0}$
(B) grad $\vec{F} = \vec{0}$
(C) div $\vec{F} = 0$
(D) curl(grad $\vec{F}$) $= \vec{0}$

Q.No.17.
The residue of $\frac{\sin z}{z^8}$ at $z = 0$ is

(A) $1/7!$
(B) $-1/7!$
(C) $1/8!$
(D) $-1/8!$

Q.No.18.
In a group G, if $a^2 = e$ then

(A) G is an abelian group
(B) G is non abelian group
(C) Ring
(D) None of these

Q.No.19.
If H is a normal subgroup of a finite group G, then number of distinct right cosets of H in G is

(A) $O(G)/O(H)$
(B) $O(G) \times O(H)$
(C) $O(G \cap H)/O(H)$
(D) None of these

Q.No.20.
The last two digits of $7^{81}$ are

(A) 07
(B) 17
(C) 37
(D) 47

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Q.No.21.
If $\vec{V}$ is the velocity of a fluid particle then $\oint_C \vec{V}.d\vec{r}$ represents

(A) Work done
(B) Circulation
(C) Flux
(D) Conservative field

Q.No.22.
The basic optimal solution set of an LPP is

(A) Either singleton or infinite
(B) Convex
(C) Finite
(D) None of these

Q.No.23.
The power series $\sum_{n=0}^{\infty} 2^{-n} z^{2n}$ converges, if radius of convergence

(A) $\sqrt{2}$
(B) $\sqrt{3}$
(C) $\infty$
(D) None

Q.No.24.
If $AX = O$ is a system of homogeneous linear equation, where A is an upper triangular matrix whose diagonal elements are 0, 1, 2, then the system of linear equation has

(A) No solution
(B) Trivial solutions
(C) Two solutions
(D) Infinite solutions

Q.No.25.
The complete solution of the differential equation $z = px + qy + p^2 + q^2$ is

(A) $ax - by + a^2 - b^2$
(B) $ax + by - a^2 - b^2$
(C) $ax + by + a^2 + b^2$
(D) None of these

Q.No.26.
$\int_{-\infty}^{\infty} f(x)\,dx$ is always equal to

(A) Zero
(B) One
(C) $f(x)$
(D) $f(x) + 1$

Q.No.27.
Joint probability of independent events $J$ and $K$ is equal to

(A) $P(J) * P(K)$
(B) $P(J) + P(K)$
(C) $P(J) * P(K) + P(J - K)$
(D) $P(J) * P(K) - P(J * K)$

Q.No.28.
Bayes rule be used in

(A) Solving queries
(B) Increasing complexity
(C) Decreasing complexity
(D) Answering probabilistic query

Q.No.29.
The solution of the differential equation $\frac{dx}{dt} + \frac{2x}{t} = 1$ is

(A) $x = C_1 + C_2 t$
(B) $x = C_1 t - \frac{1}{t}$
(C) $x = \frac{C_1}{t} + \frac{t^2}{2}$
(D) $x = \frac{C_1}{t^2} + \frac{t}{3}$

Q.No.30.
Which of the following matrices is/are positive definite?

(i) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}$ (iv) $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$

(A) (i) and (ii)
(B) (i), (ii) and (iv)
(C) (i) and (iii)
(D) (ii) and (iv)

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Q.No.31.
The coefficient of $(y-1)^2$ in Taylor's series expansion of$x^3+xy^2$ about $(2,1)$ is

(A) 1
(B) 0
(C) ½
(D) 2

Q.No.32.
The value of m so that $2x-x^2+my^2$satisfies Laplace's equation will be

(A) 1
(B) 2
(C) 3
(D) 4

Q.No.33.
A division ring is

(A) Field
(B) Integral domain
(C) A ring with division as one operation
(D) None of these

Q.No.34.
Which of the following is a field structure?

(A) Set of all natural numbers
(B) Set of all integers
(C) Set of all irrational numbers
(D) Set of all complex numbers

Q.No.35.
Rank of the matrix A = $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 0 \\ 1 & 0 & 0 & 0 \\ 4 & 0 & 3 & 0 \end{bmatrix}$ is

(A) 0
(B) 1
(C) 2
(D) 3

Q.No.36.
The theorem "A bounded entire function is constant" is named after which mathematician?

(A) Morera
(B) Liouville
(C) Lagrange
(D) Cauchy

Q.No.37.
If dual has an unbounded solution, primal has

(A) An unbounded solution
(B) An infeasible solution
(C) A feasible solution
(D) A bounded solution

Q.No.38.
The unit normal to the surface $x^2y+2xz=4$ at point (2, -2, 3) is

(A) $1/3\,(i+2\,j\,+2\,k)$
(B) $1/3\,(i\,-\,2j\,+\,2k)$
(C) $1/3\,(-i\,+\,2\,j\,+2\,k)$
(d) $1/3\,(i\,+\,2j\,-\,2k)$

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Q.No.39.
If $(D^2 - 3D + 2)y = \cosh x$, then the complementary function is

(A) $C_1 e^{-x} + C_2 e^{-2x}$
(B) $C_1 e^{x} + C_2 e^{2x}$
(C) $C_1 e^{\frac{-x}{3}} + C_2 e^{-\frac{2x}{3}}$
(D) $\frac{e^{2x}}{5} + \frac{e^{-\frac{4x}{7}}}{7}$

Q.No.40.
The sequence $\langle (-1)^n \rangle$ is

(A) Bounded
(B) Convergent
(C) Unbounded
(D) Divergent.

Q.No.41.
The condition for which the equations $3x + 4y + 5z = \text{a}$, $4x + 5y + 6z = \text{b}, 5x + 6y + 7z = \text{c}$, have solution is?

(A) $a + b + c = 0$
(B) $a + c = -2b$
(C) $\text{a} + \text{c} = 2\text{b}$
(D) $\text{a} + \text{c} \neq 2\text{b}$

Q.No.42.
The function $f(x) = -x^2 + 5x + 1$, where $x \in R$, is

(A) Convex
(B) Both Convex and Concave
(C) Neither concave nor convex
(D) Concave

Q. No.43.
Transcendental equation may have a

(A) Finite number of roots
(B) Finite or infinite number of roots
(C) Infinite number of roots
(D) None of these

Q. No.44.
If one root of $f(x) = 0$ is near to $x_0$ then the first approximation of this root as calculated by Newton-Raphson method is the abscissa of the point, where a straight line intersects the $x-$ axis. Identify the straight line from the following options.

(A) Normal to the curve $y = f(x)$ at the point $(x_0, y = f(x_0))$
(B) Passing through the point $\big(x_0, y = f(x_0)\big)$
(C) Straight line through the point $(x_0, y = f(x_0))$ having the gradient $\frac{1}{f'(x)}$
(D) Tangent to the curve $y = f(x)$ at the point $(x_0, y = f(x_0))$

Q.No.45.
Which of the following is not a metric over R?

(A) $d(x, y) = |x^2 - y^2|$
(B) $d(x, y) = |x - y|$
(C) $d(x, y) = \left|\log(\frac{x}{y})\right|$
(D) None of these

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